3D FFT MODE FILTERING FOR PHASE GRADIENT VELOCITY MEASUREMENTS

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INTRODUCTION

Palpation is a diagnostic tool that is commonly used by physicians to assess the elasticity of tissues. A medical imaging technique capable of quantitatively mapping tissue elasticity is of clinical interest because disease can affect the elastic properties of tissue [1]. Magnetic Resonance Elastography (MRE) [2, 3] is currently being developed to quantitatively image the elastic properties of tissue. MRE measures the displacements caused by a shear wave propagating within tissue. From the MRE displacement data, the propagation characteristics of the shear wave are determined and used to quantify elastic properties of the tissue. The MRE technique has three components: methods for generating acoustic shear waves, magnetic resonance imaging sequences capable of imaging cyclic displacements, and image processing techniques to determine phase velocity. The focus of this work has been to develop robust image processing techniques that will accurately relate measured shear velocity to tissue elasticity.

When processing the MRE cine images it is difficult to separate the different modes of tissue motion: vibrations, guided waves, shear waves, reflections, and ambient background motion. The other modes of motion will adversely affect the measurement of shear wave velocities. We assert that by separating the propagating shear waves from the other modes, accurate velocity measurements can be obtained. The resulting velocities will then reflect the elastic properties of the tissue being tested and provide a better physical understanding of the tissue's response. The goals of this work are the following: to isolate the shear wave using a Mode Filter implemented with a Three Dimensional (3D) Fast Fourier Transform (FFT), to make velocity measurements on shear waves using a Phase Gradient processing algorithm and to separate complex tissue motion into simpler components.

METHODS

The determination of material properties from propagating wave characteristics [4-6] is a more tractable technique than determining material properties from resonant vibrations [7]. Therefore, the desired mode of motion to isolate and analyze is the propagating shear wave. A simple model for the propagating shear wave [5] is a plane wave in a homogeneous, isotropic, and infinite material. The other modes of motion such as reflections, vibrations [6] and guided waves [5] interfere with the phase velocity measurement of the propagating wave. Table I shows the displacement models used to understand and separate multiple modes of tissue motion. Since the tissue specimens are being excited in the 100 to 1000 Hz range, all of the modes listed in Table I are likely to contribute to the MRE displacement images.

The displacement data, obtained numerically from the displacement models and experimentally from MRE, contain 3D (two spatial and one temporal) information. The acquired data consists of 2D spatial maps of displacement which are obtained synchronously in time with respect to the excitation frequency [2, 3]. Each spatial map, in a cine acquisition set of eight images, represents a sequential "snap shot" of the tissue's displacement at a specific phase in an excitation cycle. This displacement data is stored in computer memory as a 3D matrix for processing.

Table I. Displacement models used for developing the 3D FFT mode filtering algorithm.

$d_l^p(x,y,t) = A_f\cos(k_x^f x + k_y^f y - \omega_e t) + A_r\cos(k_x^r x + k_y^r y - \omega_e t)$ Forward and Reflected Shear Waves
$d_l^v(x,y,t) = B\sin(k_x^v x)\sin(k_y^v y)\cos(\omega_{nm} t)$ Membrane Vibration with Fixed Edges
$d_l^g(x,y,t) = \left(C_1\sin(k_y^g y) + C_2\cos(k_y^g y)\right)\cos(k_x^g x - \omega_e t)$ Guided Waves in Thin Plates

Where:

A_f = Amplitude of the forward wave	k_x^f = Forward wave number in x direction
A_r = Amplitude of the reflected wave	k_y^f = Forward wave number in y direction
a = Width of the membrane	k_y^g = $j\pi/(2b_t)$ $\quad j = 0,1,2,3,\ldots$
B = Displacement amplitude of the vibration	k_x^g = $[\omega_e^2/c_s^2 - (k_y^g)^2]^{0.5}$
b = Height of the membrane	k_x^r = Reflected wave number in x direction
$2b_t$ = Plate thickness	k_y^r = Reflected wave number in y direction
C_1 = Amplitude of the antisymmetric plate mode	k_x^v = $n\pi/a$ $\quad n = 1,2,3,\ldots$
C_2 = Amplitude of the symmetric plate mode	k_y^v = $m\pi/b$ $\quad m = 1,2,3,\ldots$
c_s = Shear wave phase velocity	t = Time
d_l^g = Guided wave displacement in the z direction	ω_e = Excitation frequency
d_l^p = Displacement from propagating waves	$\omega_{nm} = \pi c_s\sqrt{\left(\frac{n}{a}\right)^2 + \left(\frac{m}{b}\right)^2}$
d_l^v = Vibrational displacement	

The confounding modes are filtered by isolating the desired propagating wave using 3D FFT Mode Filtering. The desired mode of motion is extracted by choosing the appropriate spatial and temporal frequencies [4]. Since the displacement models shown in Table I are based upon sinusoidal functions, the 3D FFT of these models produce Kronecker delta functions through out temporal and spatial frequency space as shown in Figure 1. Figure 1 displays the graphical representation of the 3D magnitude spectra formed by the models of tissue motion. The 3D magnitude spectra are obtained by taking the 3D FFT of the displacements generated by the equations in Table I. The spatial frequency components consist of Kronecker delta functions in the temporal frequency planes at $\pm\omega_e$. The number of frequency components and spectral symmetry depend on the type of motion present. Each mode of tissue motion has its own unique spectral symmetry, which is used to extract the desired mode. These modes can only be separated when their respective spatial frequency components are distinct as shown.

Phase velocity measurements are made by taking the temporal FFT of the 3D displacement data [8, 9]. This converts the time series of the 2D spatial maps into temporal harmonic images of displacement magnitude and phase. The magnitude image of the fundamental temporal harmonic is a spatial map of the displacement amplitudes caused by the cyclic excitation in the tissue. The magnitude image can be used to determine attenuation in the tissue [10]. The phase image of the fundamental temporal harmonic is a spatial map of the phase progression resulting from the tissue displacement. Phase progression is inversely proportional to phase velocity [5, 8].

The 3D FFT Mode Filtering and Phase Gradient Velocity processing techniques have been tested using numerical simulation and displacement data from MRE cine acquisition. The numerical simulations of tissue displacements are generated by the linear superposition of three modes: Forward propagating wave, Reflected wave, and Vibration. The forward and reflected propagating waves were modeled as plane waves with attenuation [5] and have a phase velocity of 8.0 m/s. The vibrational component of the tissue was modeled as a single mode (n = 4, m = 7) from a free vibration of a rectangular membrane with a fixed rim [6]. The frequency of excitation for all the modes is 530 Hz. Random noise is also added to the displacement data.

The tissue simulating phantom used in the MRE acquisition of displacement data consists of two gel blocks with an oblique interface. The upper block consists of a more rigid gel (gel 1) than the lower block (gel 2). Shear waves at 350 Hz were applied to the top surface by a contact plate connected to an electromechanical actuator. Fresh porcine skeletal muscle is used for *ex vivo* tissue testing at physiologic temperature. To vary the boundary conditions imposed on the muscle tissue, the tissue is immersed in a semi-solid solution consisting of cornstarch and water and in air.

RESULTS AND DISCUSSION

The extraction of the forward propagating wave from a simulated reflection and vibration by the 3D Mode Filter is shown in Figure 2. Figure 2(a) shows a representative "original cine image" that contains the superposition of displacements caused by the multiple modes of motion. The shear wave that is forward propagating is traveling from the top to the bottom of the image. The reflected shear wave is propagating from the bottom up. The vibration affects the entire field of view with a checkerboard pattern. Figure 2(b) shows the extracted forward propagating wave which has been mode filtered from the confounding modes. Figure 2(c) consists of the "remaining modes" and

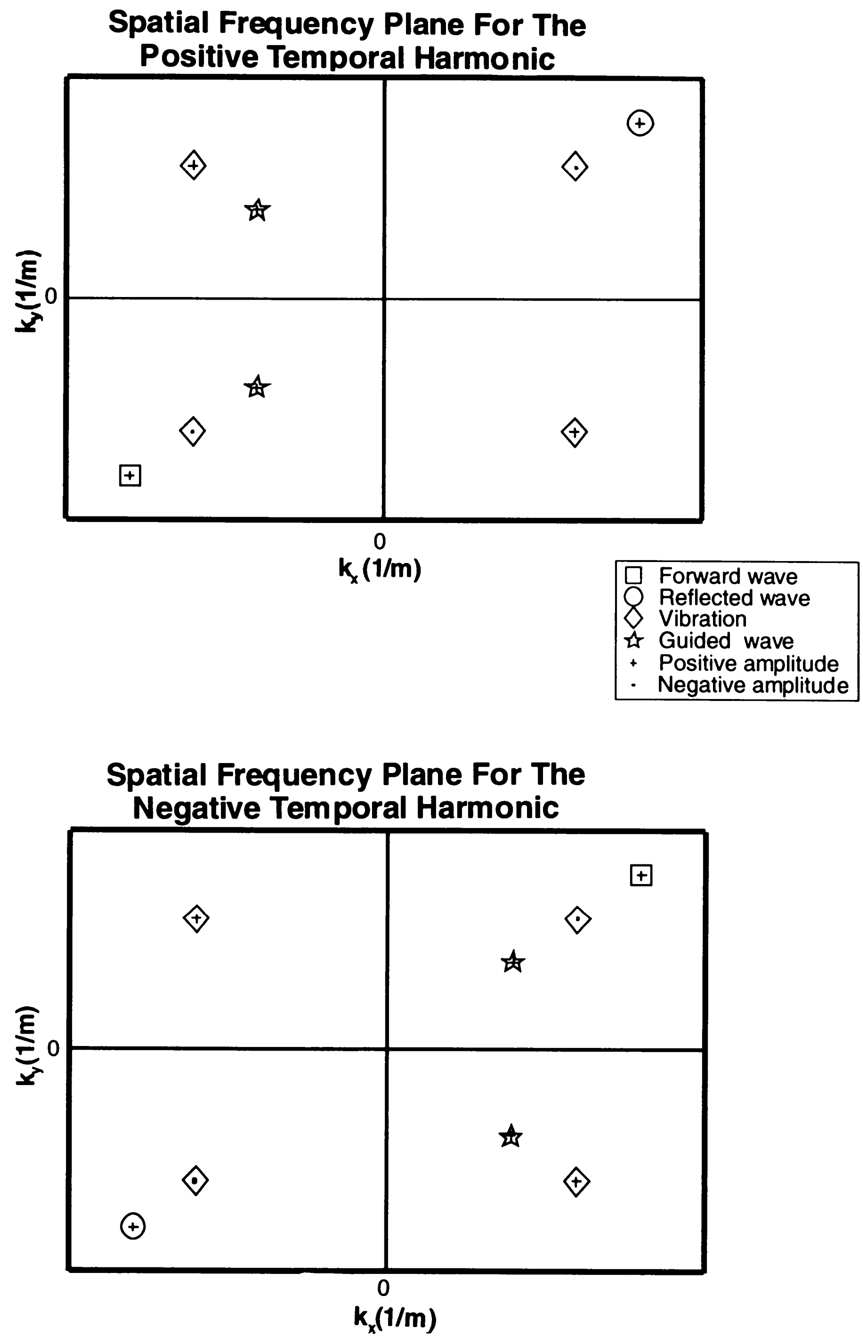

Figure 1. Graphical representation of the 3D FFT magnitude spectra formed by the modes of tissue motion given in Table I.

contains the displacements caused by the reflected and vibrational modes along with most of the noise.

Although the resulting displacement field from motion due to multiple modes is a linear superposition of the component modes, the resulting phase relationship in the displacement field becomes nonlinear. Thus multiple modes will adversely affect the linear phase progression of propagating waves. This nonlinear effect on phase progression is demonstrated by making velocity measurements on the unfiltered displacement images shown in Figure 2(a). The unfiltered velocity measurement is 83.5 m/s with a standard deviation greater than 85 m/s. The unfiltered velocity value is much higher than the expected value of 8.0 m/s for the propagating shear wave. This is contrasted with measured phase velocity of the extracted wave shown in Figure 2(b). The extracted (shear wave) velocity value is 8.00 m/s with a standard deviation of 0.006 m/s within the same region of interest as the unfiltered velocity measurement.

The results from mode filtering multiple modes of motion in a phantom are shown in Figure 3. Figure 3(a) displays a representative "original cine image" which shows an incident shear wave within a rigid agar slab (1) and a refracted shear wave with interference effects within a soft agar slab (2). The incident shear wave is traveling from the top of the image towards the bottom. The geometry of the phantom is shown in an inset within Figure 3(b). Figure 3(b) displays the displacement image of the incident shear wave which has been extracted. Figure 3(c) displays the "remaining modes" which consist of the refracted wave, interference effects, and noise. The unfiltered phase velocity measurement in the upper agar block is 6.7 m/s with a standard deviation of 0.5 m/s. The filtered phase velocity measurements in the upper agar block is 6.7 m/s with a standard deviation of 0.1 m/s. Although the same phase velocity value is measured in both the filtered and unfiltered displacement images, the velocity deviation is five times lower in the filtered images.

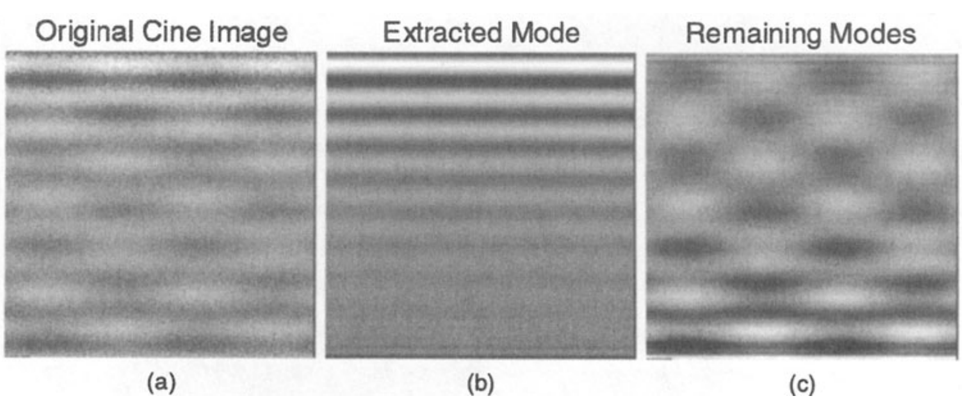

Figure 2. The extraction of a propagating wave from a reflection and a vibration by the 3D Mode Filter in simulated images. (a) An image containing the superposition of displacements. (b) The "extracted mode" image contains the forward propagating wave. (c) The "remaining modes" image contains the reflected mode, vibrational mode and noise.

A demonstration of the effects of boundary conditions on porcine tissue displacements is shown in Figure 4. Amplitude and phase images from phase gradient processing are shown for tissue immersed in a semi-solid solution and in air. In both cases, a contact plate displaces the top of the tissue. In the amplitude images, the contours of zero displacements (black) are indicative of vibrational nodes or interference. There are fewer nodes in the tissue immersed in the semi-solid solution. The phase image of the tissue immersed in the semi-solid solution shows that the resulting phase progression starts out with linear and approximately planar phase fronts. The abrupt light to dark pixel transitions in the two phase images are 2π phase wraps.

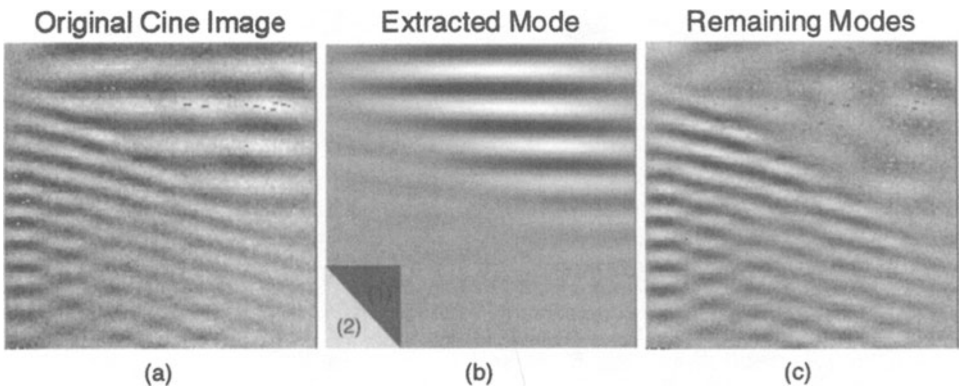

Figure 3. The extraction of an incident shear wave within a rigid agar slab (1) from the refracted shear wave and interference in a softer agar slab (2). (a) A displacement image showing refraction and interference in the gel phantom. (b) The "extracted mode" image contains the incident shear wave and an inset of the phantom geometry. (c) The "remaining modes" image consists of the refracted wave, interference patterns, and noise.

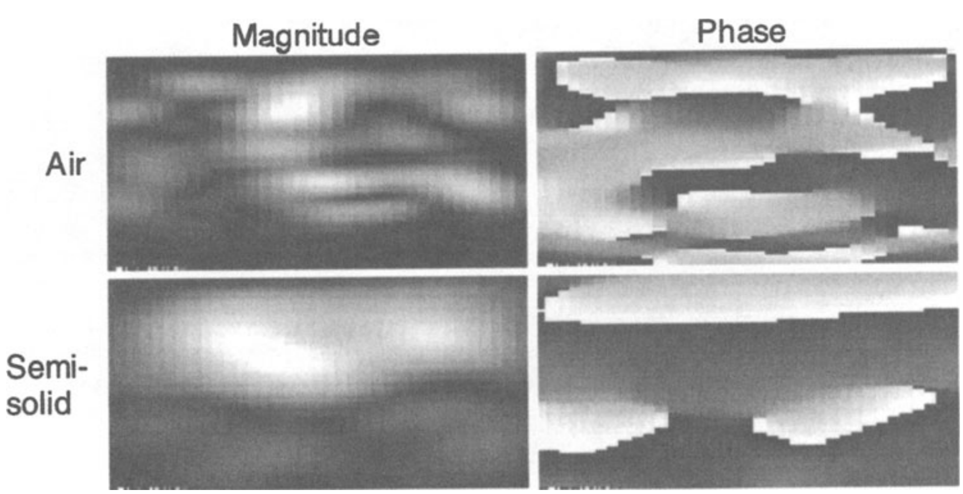

Figure 4. Demonstration of boundary condition effects on porcine tissue displacements. Amplitude and phase images from phase gradient processing are shown for tissue immersed in a semi-solid solution and in air.

CONCLUSIONS

The 3D FFT mode filter can be used recursively to isolate and reduce the effects of multiple modes in tissue motion. When the mode filter is unable to adequately isolate the propagating wave, the tissue boundary conditions can be adjusted to minimize vibrational displacements. The use of these analysis techniques will lead to the ability to measure the elastic properties in small and nonhomogeneous tissues.

In this research, image processing techniques have been developed to accurately relate measured shear velocity to tissue elasticity. Accurate phase velocity measurements can be obtained by isolating propagating shear waves from other modes of motion such as reflected shear waves, refracted shear waves, vibrations and guided waves. The desired shear wave is isolated using a mode filter implemented with a Three Dimensional Fast Fourier Transform. The phase velocity of the isolated shear wave is measured using a phase gradient processing technique.

The preliminary results indicate that:

- A specific mode of motion can be isolated from multiple modes and from noise
- Spectral symmetry can be used to extract the desired mode
- Phase Gradient processing algorithm is appropriate to use on isolated shear waves
- Mode filtering reduces deviation in phase velocity measurements
- The processing techniques provide physical intuition
- The shear wave velocity in small and nonhomogeneous tissues can be determined

These results provide motivation for continuing to develop MRE and 3D FFT Mode Filtering as tools for noninvasively evaluating the mechanical properties of *in vivo* tissue.

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